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$$B_s(B_d) \rightarrow \gamma \nu \bar{\nu}$$

Cai-Dian Lü and Da-Xin Zhang

Physics Department, Technion- Israel Institute of Technology,
Haifa 32000, Israel.

Abstract

We study the loop induced rare decays $B_s(B_d) \rightarrow \gamma \nu \bar{\nu}$ within the standard model. Both constituent quark model and pole model are used and the branching ratios turn out to be of the orders of 10^{-8} for $B_s \rightarrow \gamma \nu \bar{\nu}$ and 10^{-9} for $B_d \rightarrow \gamma \nu \bar{\nu}$. These processes can be served to determine the decay constants of B_s and B_d .

1 Introduction

Rare decays of bottom quark [1] can be served as tests of the standard model (SM). As the observed process $b \rightarrow s\gamma$ which induced $B \rightarrow K^*\gamma$ [2] and $B \rightarrow X_s\gamma$ [3] at the hadronic level, $b \rightarrow sl^+l^-$ [4] are also studied extensively in the past both at inclusive and at exclusive levels. It was recognized that the SM prediction for the branching ratio of $B \rightarrow X_s\nu\bar{\nu}$, which is $\sim 5 \times 10^{-5}$ [5], is within one order beyond the present detectivity and this process might be observed soon [6]. Furthermore, the corresponding exclusive decay $B \rightarrow K^*\nu\bar{\nu}$ is important as an input to determine V_{ub} to an accuracy of 10% [7].

Pure leptonic decays of heavy pseudoscalar mesons into light lepton pairs are helicity suppressed, their branching ratios are [8]:

$$\begin{aligned} B(B_s \rightarrow \mu^+\mu^-) &= 1.8 \times 10^{-9}, \\ B(B_s \rightarrow e^+e^-) &= 4.2 \times 10^{-14}, \end{aligned} \tag{1}$$

which make it difficult to determine f_{B_s} from these processes. For B_d the situation is even worse due to the small CKM mixing angles. Although the processes $B_s(B_d) \rightarrow \tau^+\tau^-$ do not suffer from this suppression mechanism and the branching ratio is about 10^{-7} [5] in SM, it is hard to be detected at future B -factory where the efficiency is not better than 10^{-2} . The processes $B_s(B_d) \rightarrow \gamma\nu\bar{\nu}$, whose branching ratios depend quadratically on $f_{B_s}(f_B)$, can be taken as a possible alternate to determine the decay constants. For detections of these processes the method of searching for the “missing mass” can be used, which is similar to the case of $B \rightarrow X_s\nu\bar{\nu}$ [6].

In the present work, we study the processes $B_s(B_d) \rightarrow \gamma\nu\bar{\nu}$ within the SM. We will analyze $B_s(B_d) \rightarrow \gamma\nu\bar{\nu}$ in the following: In section 2 the relevant effective Hamiltonian will be given in SM. Constituent quark model and pole model will be used in section 3 to give the predictions. Finally, section 4 contains some brief discussion.

2 Effective Hamiltonian

Let us start with the quark level process $b \rightarrow q\nu\bar{\nu}$, with $q = s$ or d . The Feynman diagrams are displayed in Fig.1. Both box and Z penguin diagrams contribute to this process. The resulting effective Hamiltonian is left handed in SM [9, 5]:

$$\mathcal{H} = C(\bar{q}\gamma_\mu P_L b)(\bar{\nu}\gamma^\mu P_L \nu), \quad (2)$$

with $P_L = (1 - \gamma_5)/2$. The coefficient C is

$$C = \frac{\sqrt{2}G_F\alpha}{\pi\sin^2\theta_w}V_{tb}V_{tq}^*\frac{x}{8}\left(\frac{x+2}{x-1} + \frac{3x-6}{(x-1)^2}\ln x\right), \quad (3)$$

where $x = m_t^2/m_W^2$. For simplicity, we neglect the QCD correction to this coefficient, whose effects are within 2% if appropriate renormalization point is chosen [5].

Since the neutrinos are massless in minimal SM, the processes $B_q(b\bar{q}) \rightarrow \nu\bar{\nu}$ ¹ are forbidden by helicity. If an additional photon line is attached to any of the charged lines in Fig.1, the situation will be different: no helicity suppression exists any more. However, when the photon line is attached to one of the internal charged lines, there will be a suppression factor of m_b^2/m_W^2 in the Wilson coefficient compared with the ones for $b \rightarrow q\nu\bar{\nu}$. The reason is that these effective operators are now dimension-8 instead of dimension-6. On the other hand, the diagrams in Fig. 1 with photon line connected to one of the external (bottom or strange quark) lines, whose effective Hamiltonian turn out to be

$$\mathcal{H} = -\frac{e}{6}C\bar{q}\left[\not{\epsilon}_\gamma\frac{\not{p}_\gamma - \not{p}_q + m_q}{(p_q \cdot p_\gamma)}\gamma_\mu P_L + P_R\gamma_\mu\frac{\not{p}_b - \not{p}_\gamma + m_b}{(p_b \cdot p_\gamma)}\not{\epsilon}_\gamma\right]b(\bar{\nu}\gamma^\mu P_L \nu), \quad (4)$$

will be the dominant contributions to the decay $B_q(b\bar{q}) \rightarrow \gamma\nu\bar{\nu}$.

3 Model calculations

The effective Hamiltonian given in (4) is not enough to analyze the processes $B_q \rightarrow \gamma\nu\bar{\nu}$. In addition, models are needed to do the calculations at the hadronic level. First we use a simple

¹We denote mesons with the quark content $(b\bar{q})$ as B_q for convenience.

constituent quark model (see, for example [11]). In this model both of the (anti)quarks are treated non-relativistically with the same velocity of the hadron. Thus (anti)quarks masses are now constituent quark masses of the order of several hundred MeV. We use further the interpolating field technique[12] which relates all the hadronic matrix elements of the present case to the decay constants of the mesons. The amplitude for $B_q \rightarrow \gamma \nu \bar{\nu}$ decay is:

$$\mathcal{A} = \frac{eCf_{B_q}m_{B_q}}{12(p_{B_q} \cdot p_\gamma)} \left[\left(\frac{1}{m_q} + \frac{1}{m_b} \right) i\epsilon_{\alpha\beta\mu\nu} \epsilon_\gamma^\alpha p_\gamma^\beta p_{B_q}^\nu + \left(\frac{1}{m_q} - \frac{1}{m_b} \right) (p_{\gamma\mu} \epsilon_{\gamma\nu} - p_{\gamma\nu} \epsilon_{\gamma\mu}) p_{B_q}^\nu \right] (\bar{\nu} \gamma^\mu P_L \nu). \quad (5)$$

The first term inside the bracket comes from the CP-odd part of B_q , while the second term results from the CP-even part of B_q . Since $m_b \gg m_q$ ($q = d, s$), we can safely neglect the term of $1/m_b$. Then after squaring the amplitude, both CP-odd and CP-even parts give the same contribution to the decay rate. Performing the phase space integration over one of the two Dalitz variables, and summing over the three generation of neutrinos, we get the differential decay width versus the $\nu\bar{\nu}$ invariant mass:

$$\frac{d\Gamma}{dm_{\nu\bar{\nu}}^2} = \frac{2C^2\alpha f_{B_q}^2}{3(48\pi)^2 m_{B_q} m_q^2} (m_{B_q}^2 - m_{\nu\bar{\nu}}^2) m_{\nu\bar{\nu}}^2. \quad (6)$$

The decay width is:

$$\Gamma = \frac{C^2\alpha f_{B_q}^2 m_{B_q}^5}{(144\pi)^2 m_q^2}. \quad (7)$$

Using $\alpha = 1/132$, $m_s = 0.51$ GeV, $m_t = 176$ GeV and $|V_{tb}V_{ts}^*| = 0.04$, we get

$$\Gamma(B_s \rightarrow \gamma \nu \bar{\nu}) = 9.5 \times 10^{-21} \times \left(\frac{f_{B_s}}{0.2 \text{ GeV}} \right)^2 \text{ GeV}. \quad (8)$$

If the lifetime is taken as $\tau(B_s) = 1.34 \times 10^{-12} \text{ s}$ [10], the branching ratio is found to be 1.9×10^{-8} . The differential decay rate versus $m_{\nu\bar{\nu}}^2$ is displayed in Fig.2 (solid line).

For B_d meson decay, we take $|V_{tb}V_{td}^*| = 0.01$ and $m_d = 0.35$ GeV. The decay width is then

$$\Gamma(B_d \rightarrow \gamma \nu \bar{\nu}) = 1.2 \times 10^{-21} \times \left(\frac{f_{B_d}}{0.2 \text{ GeV}} \right)^2 \text{ GeV}, \quad (9)$$

which corresponds to the branching ratio 2.6×10^{-9} , if uses of $\tau(B_d) = 1.50 \times 10^{-12} \text{ s}$ [10] and $f_{B_d} = 0.2 \text{ GeV}$ are made.

We also apply the pole model to calculate the B_q meson decay. The main contribution here is the radiative transition of the B_q meson into an intermediate (virtual) B_q^* . The Feynman diagram is shown in Fig.3. The subsequent process $B_q^* \rightarrow \nu\bar{\nu}$ is determined in SM by the effective Hamiltonian eqn.(4), while the $B_q B_q^* \gamma$ transition is taken as

$$\mathcal{L} = g\epsilon_{\nu\lambda\alpha\beta}\epsilon_\gamma^\nu p_\gamma^\lambda p_{B_q}^\alpha \epsilon_{B_q^*}^\beta + \text{h.c.} \quad (10)$$

This corresponds to the first part of eqn.(5), which is for the CP-odd part of B_q . Hereafter, we will concentrate only on the CP-odd combination of B_q and \bar{B}_q . The effective coupling constant is usually explained as a function of the invariant mass of the intermediate B_q^* . Here we simply derive its value from the constituent quark model as a constant [11],

$$g = -\frac{e}{3} \left(\frac{1}{m_b} + \frac{1}{m_q} \right). \quad (11)$$

The $1/m_b$ term can also be neglected compared to the $1/m_q$ term. Note that the potential model[13] gives

$$g = -\frac{e}{3} \left(\frac{1}{\Lambda_b} + \frac{1}{\Lambda_q} \right), \quad (12)$$

with

$$\begin{aligned} \Lambda_d &= 0.59\text{GeV}, \quad \Lambda_b = 4.93\text{GeV} \quad (\text{for } B_d), \\ \Lambda_s &= 0.66\text{GeV}, \quad \Lambda_b = 4.98\text{GeV} \quad (\text{for } B_s), \end{aligned} \quad (13)$$

which is quite close to (11). Now the amplitude of the decay $B_q \rightarrow \gamma\nu\bar{\nu}$ is:

$$A = \frac{C g f_{B_q^*} m_{B_q^*}}{\sqrt{2}(m_{B_q^*}^2 - m_{\nu\bar{\nu}}^2)} \epsilon_{\nu\lambda\alpha\beta} \epsilon_\gamma^\nu p_\gamma^\lambda p_{B_q}^\alpha (\bar{\nu} \gamma^\mu P_L \nu). \quad (14)$$

Here again we give the differential cross section for the purpose of being used experimentally:

$$\frac{d\Gamma}{dm_{\nu\bar{\nu}}^2} = \frac{2C^2 \alpha f_{B_q^*}^2 m_{B_q^*}^2}{3(48\pi)^2 m_{B_q}^3 m_q^2} \frac{(m_{B_q}^2 - m_{\nu\bar{\nu}}^2)^3 m_{\nu\bar{\nu}}^2}{(m_{B_q^*}^2 - m_{\nu\bar{\nu}}^2)^2}. \quad (15)$$

After integrating over the phase space, the decay width is obtained as

$$\Gamma = \frac{C^2 \alpha f_{B_q^*}^2 m_{B_q^*}^8}{(144\pi)^2 m_{B_q}^3 m_q^2} f(m_{B_q}^2/m_{B_q^*}^2), \quad (16)$$

where

$$f(y) = -5y^3 + 6y^2 + 12y - 12 - 6(4-y)(1-y)^2 \ln(1-y).$$

The formula (15,16) are similar to the constituent quark model case eqn.(6,7). It is easy to see that if $m_{B_q^*} = m_{B_q}$, $f_{B_q^*} = f_{B_q}$ are taken in eqn.(15,16), they will reduce exactly to eqn.(6,7).

Now we use $m_{B_s^*} = 5.42$ GeV and get:

$$\Gamma(B_s \rightarrow \gamma \nu \bar{\nu}) = 8.8 \times 10^{-21} \times \left(\frac{f_{B_s^*}}{0.2 \text{ GeV}} \right)^2 \text{ GeV}. \quad (17)$$

It is only slightly different from eqn.(8), because $m_{B_s^*}$ is not quite different from m_{B_s} . The branching ratio is 1.8×10^{-8} , if $f_{B_s^*} = f_{B_s} = 0.2$ GeV is used. The differential decay rate calculated in pole model is also displayed in Fig.2 (dashed line) as function of $m_{\nu\bar{\nu}}^2$.

For B_d meson, we get

$$\Gamma(B_d \rightarrow \gamma \nu \bar{\nu}) = 1.1 \times 10^{-21} \times \left(\frac{f_{B_d^*}}{0.2 \text{ GeV}} \right)^2 \text{ GeV}. \quad (18)$$

The branching ratio is obtained as 2.4×10^{-9} , also quite close to that obtained in constituent quark model.

4 Conclusion

We predict the branching ratios in SM for $B_s \rightarrow \gamma \nu \bar{\nu}$ to be 10^{-8} and for $B_d \rightarrow \gamma \nu \bar{\nu}$ to be 10^{-9} . With these branching ratios, they are hopeful to be detected at future B factories or LHC. They can provide alternate channels for measuring f_{B_s} (f_{B_d}).

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Figure Captions

Fig.1 Feynman diagrams in standard model for $b \rightarrow q\nu\bar{\nu}$.

Fig.2 Differential decay rate of $B_s \rightarrow \gamma\nu\bar{\nu}$ versus the $\nu\bar{\nu}$ invariant mass $m_{\nu\bar{\nu}}^2$, with the solid line denoting results calculated from constituent quark model, and the dashed line corresponding to that of pole model.

Fig.3 Feynman diagram in pole model for $B_q \rightarrow \gamma\nu\bar{\nu}$.

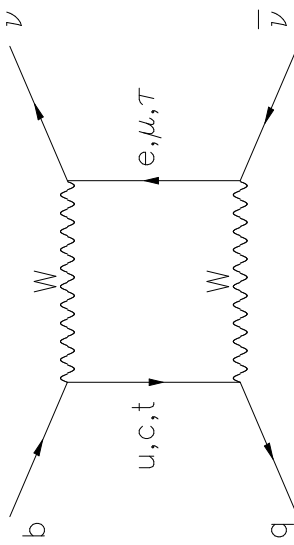
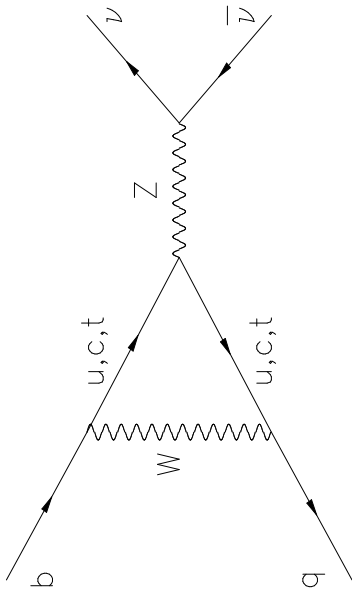
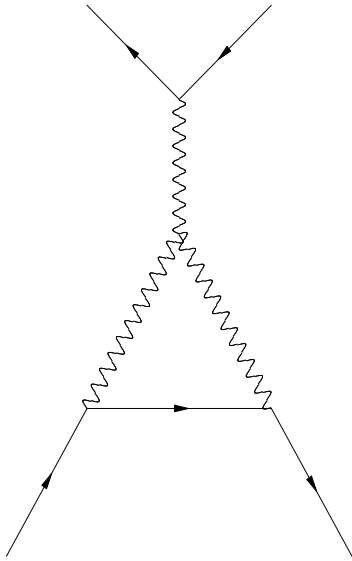


Fig. 1

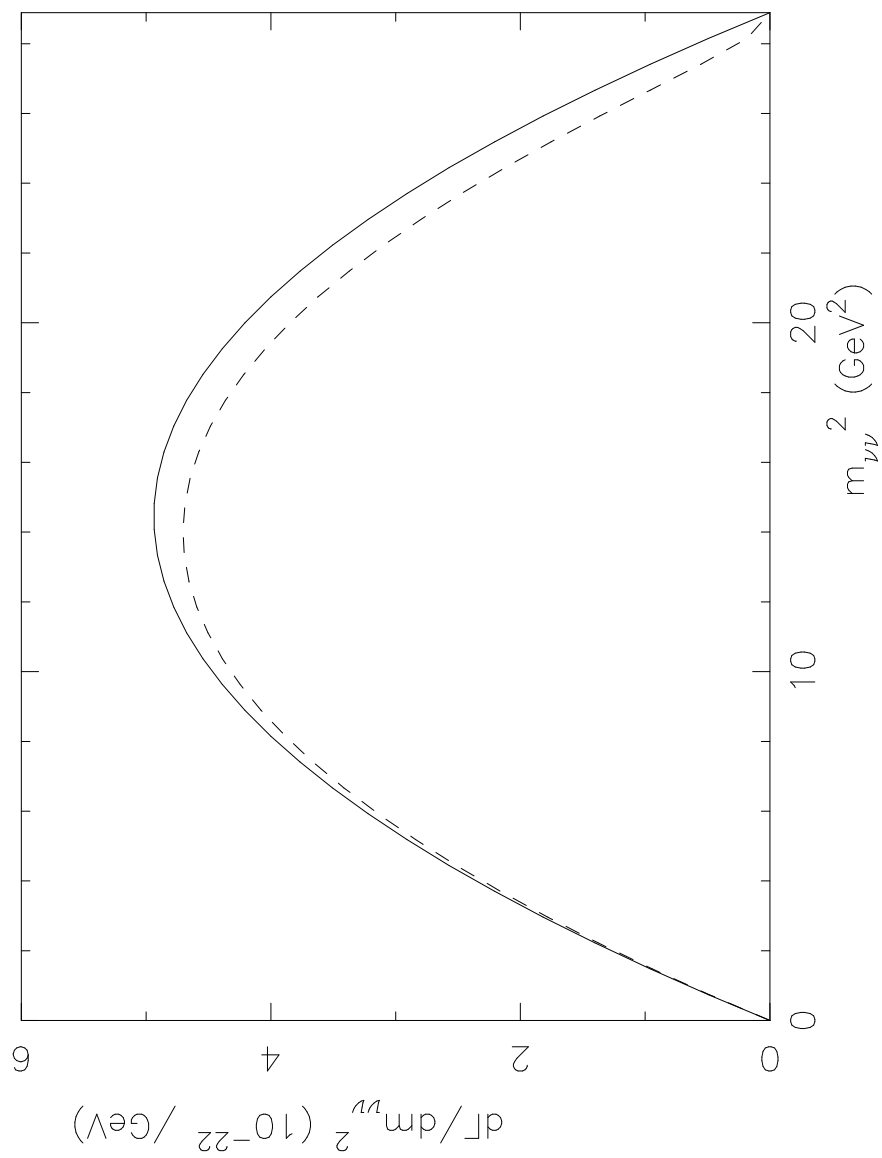


Fig. 2

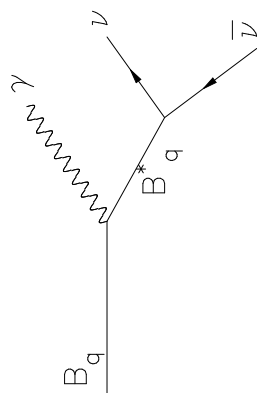


Fig. 3